Expanding lattice theory driven mathematical models to define fullerene-like viral capsid structures

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Purpose

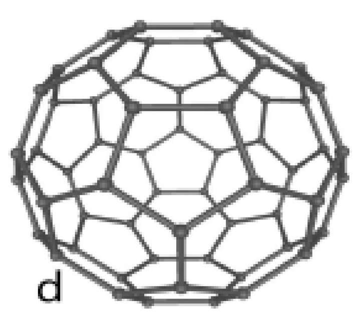
This research project expands upon the lattice theory model of viral capsids in order to establish determinants and requirements to account for the oversimplified complexity of capsid handedness. This, in turn, defines specifics of the model for a wider variety of viral capsids.

What is a viral capsid?

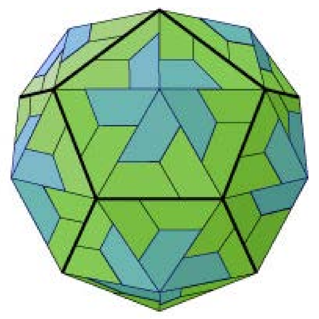
- The outer shell of a virus which protects its genetic material (DNA or RNA)
- Made up of protein subunits called capsomeres

Fullerenes and Icosahedral Capsids

- **Fullerene** – type of polyhedron, specifically a carbon molecule, in which the C atoms, i.e., the vertices of the polyhedron, are arranged as pentagons and hexagons.
- Fullerenes have a 3-valent nature – each vertex associated with only three edges.
- Many viral capsids have capsomeres in the form of 3-valent pentagons and hexagons leading them to be considered as **fullerene-like capsids [2]**.



M. Deza Fullerenes: Applications and Generalizations. Slides. Allotropes of Carbon Fig d.



T=3 capsid <http://viralzone.expasy.org/all_by_protein/806.html>

Caspar and Klug Theory

- Theory for mathematically modelling icosahedral viral capsid structure and understanding arrangement of constituent subunits.
- **T- number** related to the length of one side of the 20 triangles creating the faces of an icosahedron (20-faced polyhedron).
  - Defined as **$T = h^2 + hk + k^2$** where h and k are positive integers,
- **Quasi-equivalence** – explains distribution of capsid proteins within these triangles, when they are identical **[1]**.

Lattice Theory Models

- Build on aforementioned ideas to produce representations of fullerene-like capsids using generating vectors, hexagonal lattices and their triangular duals **[7]**.

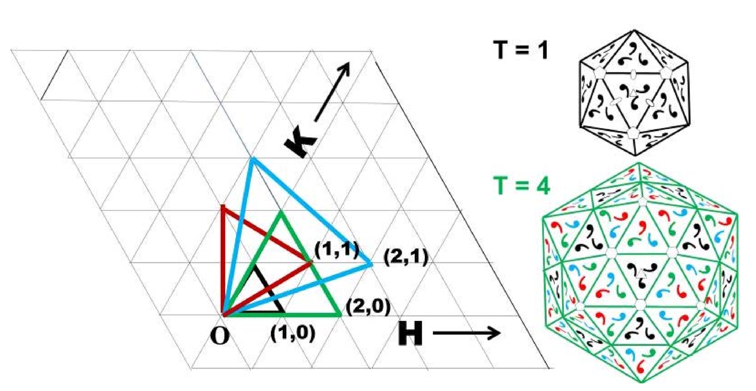


Fig. 4. Prasad et al. Principles of Virus Structural Organization

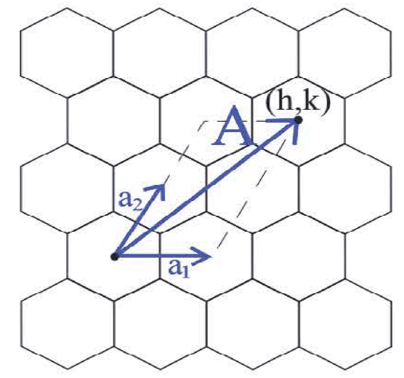
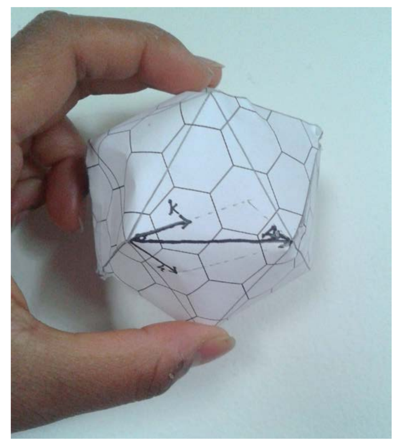
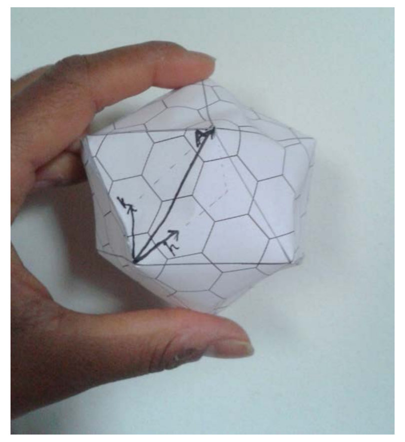


Fig. 1. Sadre-Marandi et al. Generating Vectors for the Lattice Structures of Tubular and Conical Viral Capsids

Handedness

- Occurrence of right/left handed orientation in some capsids due to existence of more than one set of (h,k) values producing the same T- number **[5]**.
- Can lattice theory model accommodate this feature?
- Ex: Viral capsid with T= 7 → (1,2) and (2,1)





Carbon Nanotubes and Chiral Angle

- CNT – cylindrical allotrope of carbon
- Three configurations – **zigzag, armchair and chiral**
- Configuration determined by chiral vector and chiral angle
- Chiral vector (n,m) of CNT equivalent to generating vector (h,k)
- Chiral angle (**θ**) of CNT equivalent to angle b/w generating vector and basis vectors (a_1 and a_2)
  - **θ** = 0°/60° [(m,0) or (n,0)] => Zigzag
  - **θ** = 30° [(n,n) or (m,m)] => Armchair
  - Chiral configuration otherwise **[9]**.

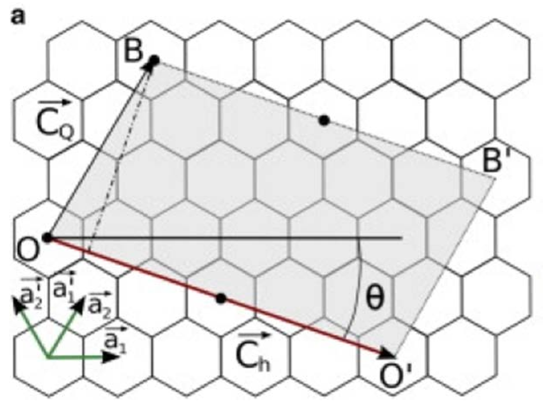


Fig. 6a A. Luque and D. Reguera, The Structure of Elongated Viral Capsids. Biophysical Journal 98 (2010). 2993-3003.

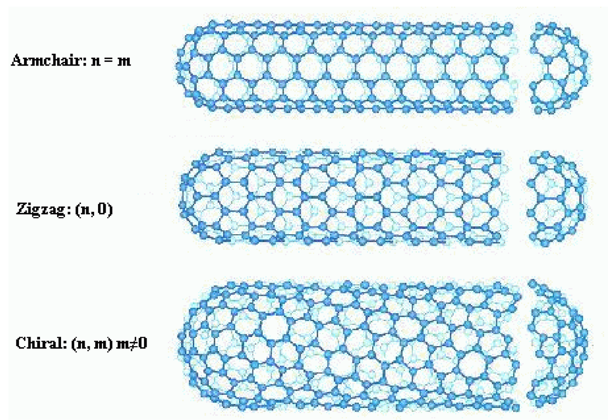
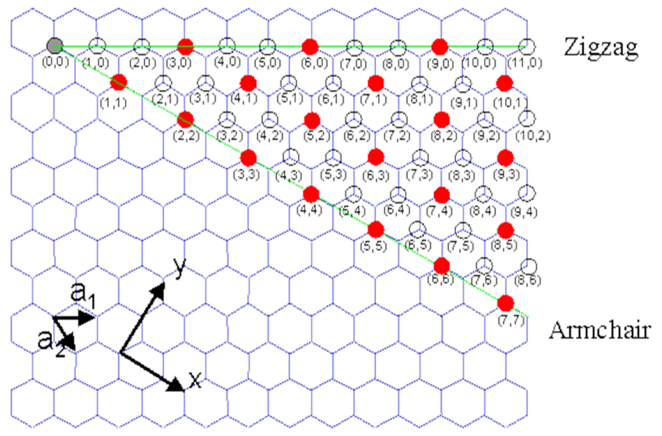


Fig. 2 J. Sinclair, An Introduction to Carbon Nanotubes. University of Tennessee. March 24 2009.



<http://www.photon.t.u-tokyo.ac.jp/~maruyama/kataura/chirality.html>

Results

- Two main requirements for handedness to occur in a viral capsid:
  - h or k ≠ 0
  - h ≠ k
- Handedness can be described using a P-number defined as shown below:
  Consider f = greatest common divisor of h & k
  Let $h=fh_0$ and $k=fk_0$ (h_0,k_0 are arbitrary values >0)
  Therefore,

$$\mathbf{T = h^2 + k^2 + hk}$$
$$= f^2h_0^2 + f^2k_0^2 + f^2h_0k_0$$
$$= (h_0^2 + k_0^2 + h_0k_0)f^2$$
$$= \mathbf{Pf^2} \quad \text{where } \mathbf{P = h_0^2 + k_0^2 + h_0k_0 = 1,3,7,\ldots}$$

- From these restrictions and established relationships, it follows that viruses with P ≥7 must have handedness:

$$\mathbf{T = (h_0^2 + k_0^2 + h_0k_0)f^2}$$

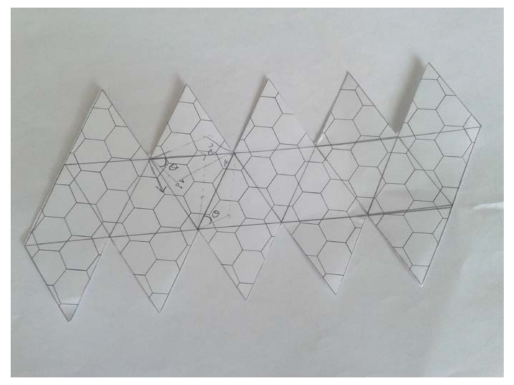
Case I: (h,k) = (0,a) => T = a^2 (1^2), **P=1** [(h,0) or (0,k)]
Case II: (h,k) = (a,a) => T = 1^2($3a^2$)= a^2 (3), **P=3** [(h,h) or (k,k)]
Case III: (h,k) = (a,b) => T = f^2(P), **P≥7**

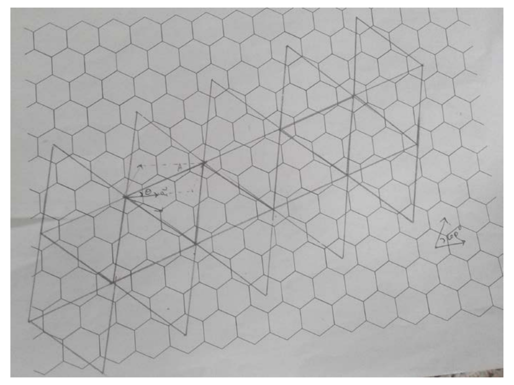
- The occurrence of handedness further determined by the chiral angle:

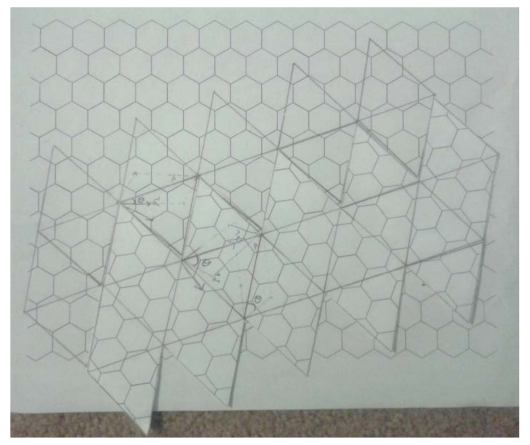
$$\boldsymbol{\theta = \cos^{-1}\left(\frac{2h+k}{2\sqrt{T}}\right)}$$

- **(h,0) or (0,k)** [0°/60°] => no handedness
- **(h,h) or (k,k)** [30°] => no handedness
- **h>k** b/w 0° & 30° => handedness
- **k>h** b/w 30° & 60° => handedness

- Type of handedness (left or right) depends on if h>k or k>h







Relevance

- More refined and thorough version of the lattice theory model obtained
- Results enhance and advance current understanding of viral capsid structures through improving and defining their specificity.
- Increases ability to predict and determine structure of viral capsid and possibly arrangement of hexamers
- Fuels further research in the same or similar areas of study
- A number of advantages of viruses have been found **[3]**.
  - gene therapy
  - virotherapy to fight cancer
  - biological pest control
  - nucleation of various chemical reactions (specifically capsids) **[4]**.

Our results and their contribution to mathematics and virology could help further such advancements in the discovery of the benefits of viruses.

References

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